High School Mathematics Contest
The departments of
MATHEMATICS and MATHEMATICS EDUCATION
EAST CAROLINA UNIVERSITY

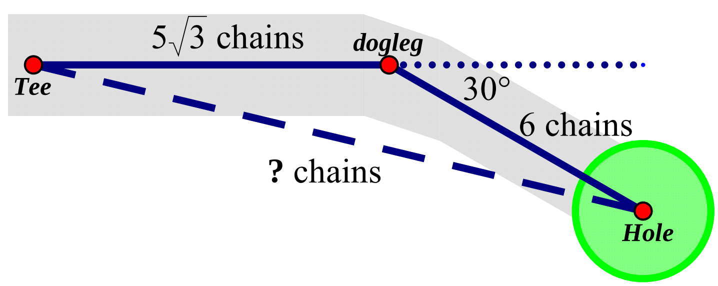
GEOMETRY: 2011

SOLUTIONS TO SELECT QUESTIONS

(N = 146)

1. A certain hole on a golf course is advertized as being 14.66 chains long (or about 330 yards). There is a 30° dogleg (a bend) as illustrated in the diagram.

   The distance from the tee to the center of the dogleg is $5\sqrt{3}$ chains, while the distance from the center of the dogleg to the hole is 6 chains.



   Find the straight-line distance from the tee to the green, in chains.

   (A) $3\sqrt{21}$ (B) $8\sqrt{3}$ (C) $\sqrt{195}$ (D) $3\sqrt{22}$ (E) $\sqrt{201}$

Correct Answer: (E)
Answer Distribution: (A) 22.6% (B) 28.1% (C) 18.5% (D) 15.8% (E) 8.2% (Other) 6.9%

Solution:
Rename Tee (point T), Dogleg (Point D), and Hole (point H). Consider ΔTDH . Let point C be the point of intersection of the altitude dropped from point H. Triangle ΔDHC is a 30-60-90 triangle with hypotenuse length $DH = 6$. Thus, $HC = 3$ and $DC = 3\sqrt{3}$. Then, triangle ΔTHC is a right triangle with hypotenuse length, which is the straight-line distance to the hole, given by

$$\sqrt{\left(5\sqrt{3}+3\sqrt{3}\right)^2+\left(3\right)^2} = \sqrt{192+9} = \sqrt{201}$$

2. A cloth has a thickness of 0.1cm. It is wound around a cardboard tube whose diameter is 5cm. The cloth wraps around the cardboard tube exactly 5 times. How long is the cloth?

(A) 25.5π cm (B) 26π cm (C) 26.5π cm (D) 27π cm (E) 27.5π cm

Correct Answer: (D)
Answer Distribution: (A) 43.8% (B) 18.5% (C) 19.2% (D) 7.5% (E) 5.5% (Other) 5.5%

Solution:
The first wrap around the tube will use $\pi d = \pi(5\text{cm})$ of material. At that point, the tube plus one wrap of cloth has diameter 5.2cm. Thus, the second wrap will use $\pi(5.2\text{cm})$ of material. Hence, the total length of cloth use will be

$$5\pi + 5.2\pi + 5.4\pi + 5.6\pi + 5.8\pi = 27\pi \, cm.$$

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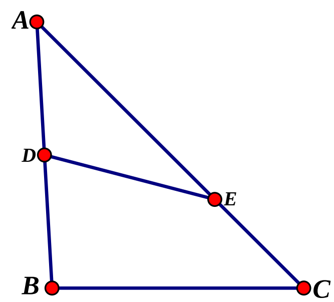
3. In $\# ABC$ point D is the midpoint of side $\overline{AB}$ and point E satisfies $AE = 2\,EC$. Determine the ratio $\text{Area}(\# ADE) / \text{Area}(\# ABC)$.



(A) $\frac{2}{5}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$

(D) $\frac{2}{3}$ (E) $\frac{3}{4}$

Correct Answer: (B)
Answer Distribution: (A) 22.6% (B) 15.1% (C) 26.7% (D) 19.9% (E) 6.9% (Other) 8.9%

Solution:

The area of a triangle is given by $Area = \frac{1}{2}(base)(height)$.

For triangles $\# ABC$ and $\# ADE$, consider For triangles $\overline{AB}$ and $\overline{AD}$ to be the bases of their respective triangles. As given in the problem, $AB = 2AD$. Now, respectively drop altitudes from points C and E to points c and e. Then triangles $\# ACc$ and $\# AEe$ are similar. Thus, the height of triangle $\# ABC$ is 1.5 times more than the height of triangle $\# ADE$. Thus,

$$\frac{\text{Area}(\# ADE)}{\text{Area}(\# ABC)} = \frac{\frac{1}{2}(AD)(Ee)}{\frac{1}{2}(AB)(Cc)} = \frac{\frac{1}{2}(AD)(Ee)}{\frac{1}{2}(2AD)(1.5Ee)} = \frac{1}{3}$$

4. The volume of a sphere is tripled. By what factor does the surface area change?

(A) $\sqrt[3]{9}$ (B) $\sqrt[3]{3}$ (C) 3

(D) 9 (E) $\sqrt{3}$

Correct Answer: (A)
Answer Distribution: (A) 6.8% (B) 24.0% (C) 22.6% (D) 26.0% (E) 7.5% (Other) 13.0%

Solution:
The formula for the volume of a sphere is $Volume = 4\pi r^3/3$. In order to triple the volume, then the radius r must be increased by a factor of $\sqrt[3]{3}$. The formula for the surface area of a sphere is $Surface\ Area = 4\pi r^2$. Thus, tripling the volume, increases the surface area by a factor of $\left(\sqrt[3]{3}\right)^2 = \sqrt[3]{9}$.

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5. A circular disc with diameter D is placed on an 8x8 checkerboard with width D so that the centers coincide. The number of checkerboard squares (out of 64) that are completely covered by the disc is

(A) 48 (B) 44 (C) 40 (D) 36 (E) 32

Correct Answer: (E)
Answer Distribution: (A) 19.2% (B) 16.4% (C) 18.5% (D) 19.2% (E) 13.7% (Other) 13.0%

Solution:
Without loss of generality, assume that each square is 1-by-1 and the entire checkerboard is 8-by-8, and the disc has radius 4. In order for the disc to completely cover a particular square, it must extend beyond the furthest corner. Consider the center of the checkerboard to be the origin. The coordinates of the 16 squares in the first quadrant and the corresponding distance of the furthest corner from the center are given by

$$\begin{array}{llll} (1,4)\to\sqrt{17} & (2,4)\to\sqrt{20} & (3,4)\to\sqrt{25} & (4,4)\to\sqrt{32} \\ (1,3)\to\sqrt{10} & (2,3)\to\sqrt{13} & (3,3)\to\sqrt{18} & (4,3)\to\sqrt{25} \\ (1,2)\to\sqrt{5} & (2,2)\to\sqrt{8} & (3,2)\to\sqrt{13} & (4,2)\to\sqrt{20} \\ (1,1)\to\sqrt{2} & (2,1)\to\sqrt{5} & (3,1)\to\sqrt{10} & (4,1)\to\sqrt{17} \end{array}$$

The disc completely covers 8 of the squares in the first quadrant. By symmetry, it also completely covers 8 squares in each of quadrants 2,3, and 4. Thus, the disc completely covers 32 squares.