

High School Mathematics Contest
The departments of
MATHEMATICS and MATHEMATICS EDUCATION
EAST CAROLINA UNIVERSITY

ALGEBRA II: 2011

SOLUTIONS TO SELECT QUESTIONS
(N = 115)

1. **How many solutions** does $\frac{x}{x^2-9} + \frac{4}{x+3} = \frac{3}{x^2-9}$ have?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Correct Answer: (A)

Answer Distribution: (A) 12.2% (B) 34.8% (C) 36.5% (D) 9.6% (E) 6.1% (Other) 0.9%

Solution:

$$\begin{aligned}\frac{x}{x^2-9} + \frac{(x-3)}{(x+3)(x-3)} &= \frac{3}{x^2-9} \\ \rightarrow \frac{2x-3}{x^2-9} &= \frac{3}{x^2-9} \\ \rightarrow 2x-3 &= 3, \text{ excluding } x \neq -3 \text{ or } 3 \\ \rightarrow x &= 3, \text{ excluding } x \neq -3 \text{ or } 3\end{aligned}$$

2. Simplify $\frac{a^{-2} - b^{-2}}{a^{-1} + b^{-1}}$.

- (A) $\frac{1}{a-b}$ (B) $a-b$ (C) $\frac{ab}{a+b}$ (D) $\frac{b-a}{ab}$ (E) none of these

Correct Answer: (D)

Answer Distribution: (A) 49.6% (B) 18.3% (C) 6.1% (D) 3.5% (E) 22.6% (Other) 0.0%

Solution:

$$\begin{aligned}\frac{a^{-2} - b^{-2}}{a^{-1} + b^{-1}} \frac{a^2 b^2}{a^2 b^2} &= \frac{b^2 - a^2}{ab^2 + a^2 b} \\ &= \frac{(b-a)(b+a)}{ab(b+a)} \\ &= \frac{b-a}{ab}\end{aligned}$$

3. **How many solutions** does $\log_3(x-2) + \log_3(x-4) = 2$ have?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Correct Answer: (B)

Answer Distribution: (A) 3.5% (B) 33.9% (C) 46.1% (D) 11.3% (E) 4.3% (Other) 0.9%

Solution:

$$\begin{aligned}\log_3(x-2) + \log_3(x-4) &= 2 \\ \text{firstly, note that } x &\text{ must be greater than 4} \\ \rightarrow \log_3(x-2)(x-4) &= 2, \\ \rightarrow 3^{\log_3(x-2)(x-4)} &= 3^2 \\ \rightarrow (x-2)(x-4) &= 9 \\ \rightarrow x^2 - 6x - 1 &= 0 \\ \rightarrow x &= \frac{6 \pm \sqrt{36 - 4(-6)(-1)}}{2(1)} \\ \rightarrow x &= 3 \pm \sqrt{3}\end{aligned}$$

4. Simplify $\frac{y^{-1} + x^{-1}}{(xy)^{-1}}$.

- (A) $x + y$ (B) $\frac{x + y}{xy}$ (C) $\frac{xy}{x + y}$ (D) -1 (E) $x - y$

Correct Answer: (A)

Answer Distribution: (A) 12.2% (B) 13.9% (C) 60.9% (D) 6.1% (E) 3.5% (Other) 3.5%

Solution:

$$\frac{y^{-1} + x^{-1}}{(xy)^{-1}} \cdot \frac{xy}{xy} = x + y$$

5. Simplify: $\sqrt[3]{\frac{a}{b} \sqrt{\frac{b}{a}} \sqrt{\frac{a}{b}}}$

- (A) $\sqrt[3]{\frac{a^4}{b^4}}$ (B) $\sqrt[12]{\frac{a}{b}}$ (C) $\sqrt[3]{\frac{a}{b}}$ (D) $\sqrt[4]{\frac{a}{b}}$ (E) $\sqrt[12]{\frac{a^5}{b^5}}$

Correct Answer: (D)

Answer Distribution: (A) 13.9% (B) 44.3% (C) 23.4% (D) 7.0% (E) 7.8% (Other) 3.5%

Solution:

$$\begin{aligned} \sqrt[3]{\frac{a}{b} \sqrt{\frac{b}{a}} \sqrt{\frac{a}{b}}} &= \sqrt[3]{\frac{a}{b} \sqrt{\frac{b}{a} \left(\frac{a}{b}\right)^{1/2}}} \\ &= \sqrt[3]{\frac{a}{b} \sqrt{\left(\frac{b}{a}\right)^{1/2}}} \\ &= \sqrt[3]{\frac{a}{b} \left(\frac{b}{a}\right)^{1/4}} \\ &= \sqrt[3]{\left(\frac{a}{b}\right)^{3/4}} \\ &= \left(\frac{a}{b}\right)^{1/4} \\ &= \sqrt[4]{\frac{a}{b}} \end{aligned}$$