

High School Mathematics Contest  
The departments of  
MATHEMATICS and MATHEMATICS EDUCATION  
EAST CAROLINA UNIVERSITY

ALGEBRA I: 2011

**SOLUTIONS TO SELECT QUESTIONS**  
(N = 107)

1. Simplify the radical expression, assume all variables are positive when they appear:

$$(5 - 3\sqrt{y})(5 + 3\sqrt{y})$$

(A)  $25 - 9y$       (B)  $25 + 9\sqrt{y}$       (C)  $25 - 9\sqrt{y}$

(D)  $9y + 25$       (E)  $25 - 6\sqrt{y} + 9y$

Correct Answer: (A)

Answer Distribution: (A) 22.4%    (B) 8.4%    (C) 60.7%    (D) 4.7%    (E) 0.9%    (Other) 2.8%

Solution:

$$\begin{aligned}(5 - 3\sqrt{y})(5 + 3\sqrt{y}) &= 25 + 15\sqrt{y} - 15\sqrt{y} - 9y \\ &= 25 - 9y\end{aligned}$$

2. The table depicts the functions  $f(x)$  and  $g(x)$ . Find  $(f \circ g)(2)$ .

- (A) 9
- (B) 2
- (C) 4
- (D) 7
- (E) 6

$x$	$f(x)$	$g(x)$
-3	2	6
-2	4	9
-1	3	7
0	2	-2
1	-1	4
2	-3	-2
3	1	-4

Correct Answer: (C)

Answer Distribution: (A) 9.3% (B) 12.1% (C) 20.6% (D) 14.0% (E) 37.4% (Other) 6.5%

Solution:  $(f \circ g)(2)$  is equivalent to  $f(g(2))$ . According to the chart,  $g(2) = -2$ . So  $f(g(2)) = f(-2)$  which equals 4 according to the chart.

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3. Determine which of the following statements is **false**.

- (A) The graph of  $f(x) = 2^x$  is always increasing
- (B) The graph of  $f(x) = \left(\frac{1}{2}\right)^x$  is always increasing
- (C) The graph of  $f(x) = 2^x$  has a y-intercept of (0,1)
- (D) The graph of  $f(x) = \left(\frac{1}{2}\right)^x$  has a y-intercept of (0,1)
- (E) All of the above statements are true

Correct Answer: (B)

Answer Distribution: (A) 12.1% (B) 24.3% (C) 18.7% (D) 6.5% (E) 33.6% (Other) 4.7%

Solution:

- (A) Since  $f(1) = 2$  and  $f(2) = 4$ , then this supports that the function may be always increasing
- (B) Since  $f(1) = \frac{1}{2}$  and  $f(2) = \frac{1}{4}$ , then the function is clearly **not** always increasing
- (C)  $f(0) = 2^0 = 1$ . So, the y-intercept is (0,1).
- (D)  $f(0) = \left(\frac{1}{2}\right)^0 = 1$ . So, the y-intercept is (0,1).
- (E) Since B is false, not all answers are true.

4. Suppose that  $w^2 - 2w = 0$ , and  $2y^2 - y = 0$ , then  $\frac{w}{y} =$

- (A)  $\frac{w^2}{4y^2}$       (B)  $\frac{4w^2}{y^2}$       (C)  $\frac{w^2}{y^2}$       (D)  $\frac{2w^2}{y^2}$       (E)  $\frac{w^2}{2y^2}$

Correct Answer: (A)

Answer Distribution: (A) 17.8%   (B) 7.5%   (C) 30.8%   (D) 15.9%   (E) 17.8%   (Other) 10.3%

Solution:

The first equation implies  $w = w^2 / 2$ . The second equation implies  $y = 2y^2$ . Thus,

$$\frac{w}{y} = \frac{w^2 / 2}{2y^2} = \frac{w^2}{4y^2}.$$

5. A square sheet of cardboard 10 inches by 10 inches is made into a box by cutting  $x$  inch squares out of each corner. Which of the following polynomials represents the volume of the box?

- (A)  $4x^3 - 20x^2 + 100x$       (B)  $4x^2 - 40x + 100$       (C)  $4x^2 - 20x + 100$   
(D)  $4x^3 - 40x^2 + 100x$       (E)  $2x^3 - 20x^2 + 100x$

Correct Answer: (D)

Answer Distribution: (A) 12.1%   (B) 29.0%   (C) 29.0%   (D) 15.9%   (E) 2.8%   (Other) 11.2%

Solution:

The base of the box will be  $10 - 2x$  and the height will be  $x$ . Thus the volume is

$$V = (10 - 2x)^2 (x) = 100x - 40x^2 + 4x^3$$